

A relativistic dynamical model of πN scattering *

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We present a unitary relativistic quasi-potential model for describing the low-energy πN interaction, based on the equal time Bethe-Salpeter equation. It preserves the covariant structure of a relativistic spin 1/2 particle for the nucleon propagator, to be contrasted to other quasi-potential approximations.

1. INTRODUCTION

In the description of dynamical models of the πN system, the corresponding amplitude is determined as a solution of a scattering equation of the Bethe-Salpeter (BS) or Lippman-Schwinger type. The BS equation in relativistic studies is usually approximated by a covariant 3-dimensional quasi-potential (QP) reduction. In this framework one in principle has a lot of freedom, since there is no unique scheme for the choice of the equation. Certainly, one would like to restrict this freedom not only by fitting to experimental data, some restrictions can come from various symmetries and consistency requirements, such as the correct low-energy limit, the correct one-body limit of the equation [1], etc.

Another consistency requirement is the symmetry of the renormalization of the positive and negative energy-states. We find that most of the relativistic QP equations used in practice do *not* satisfy this requirement, suggesting that relativistic covariance is violated in these QP formulations. One particular choice however, the *equal-time* (ET) (or instantaneous) *approximation*[2] of the BS equation, does not suffer from this pathology.

In the next section we consider the self-energy calculation and demonstrate how the equivalence of the positive and negative energy-state renormalization can be destroyed. In section 3 we briefly present our model for pion-nucleon scattering based on the solution of the BS equation in the equal-time approximation.

2. SELF-ENERGY AND RENORMALIZATION

Consider the dressed nucleon propagator given by

$$S(\not{P}) = [\not{P} - m - \Sigma(\not{P}) + i\epsilon]^{-1}, \quad (1)$$

where $\Sigma(\not{P})$ is the self-energy. Relativistic covariance under general Lorentz transformations requires that

$$\Sigma(\not{P}) = A(P^2) \not{P} + B(P^2). \quad (2)$$

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In the c.m. frame $P = (P_0, \vec{0})$ the Dirac structure of the self-energy simplifies to $\Sigma(P_0) = \Sigma_+(P_0)\gamma_+ + \Sigma_-(P_0)\gamma_-$, where $\gamma_{\pm} = \frac{1}{2}(I \pm \gamma_0)$. A similar decomposition holds for the propagator,

$$S(P_0) = S^{(+)}(P_0)\gamma_+ + S^{(-)}(P_0)\gamma_-, \quad (3)$$

with $S^{(\pm)}(P_0) = \pm[P_0 \pm (-m - \Sigma_{\pm}(P_0) + i\epsilon)]^{-1}$. Obviously, $S^{(+)}$ corresponds to the positive and $S^{(-)}$ to the negative energy-state propagations. Since eq. (3) describes a relativistic spin 1/2 propagation, it has to have poles at $P_0 = \pm m$ with the same field renormalization constant Z_2 . This implies that near these poles we have to satisfy the condition

$$\Sigma_r(P_0) = \Sigma_{-r}(-P_0), \quad r = \pm 1. \quad (4)$$

It is easy to see that eq. (4) is valid as long as the self-energy can be written in the covariant form (2). If eq. (4) does not hold, the positive and negative energy-states have different renormalized masses and the standard renormalization procedure[3] breaks down.

Let us now study as a specific example the lowest order πN bubble self-energy graph. This occurs in the dynamical models under consideration. We have after the partial-wave decomposition:

$$\Sigma_r(P_0) = \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \int_0^{\infty} \frac{dk}{2\pi} k^2 \sum_{\rho} G^{(\rho)}(\mathbf{k}, k_0; P_0) \Phi_r^{(\rho)}(\mathbf{k}, k_0; P_0), \quad (5)$$

where $G^{(\rho)}$ is the pion-nucleon propagator and $\Phi_r^{(\rho)}$ represents the πNN vertex contributions with ρ characterizing the ρ -spin[2] of the intermediate state. Furthermore, the integration variable k_0 is the relative-energy variable, defined as $k_0 = (p_{N0} \hat{\omega} - p_{\pi 0} \hat{E})/P_0$, where p_{N0} ($p_{\pi 0}$) is the 0-th component of the nucleon (pion) 4-momentum, and $\hat{\omega} = (P_0^2 - m_N^2 + m_{\pi}^2)/2P_0 = P_0 - \hat{E}$.

The singularities of $\Phi_r^{(\rho)}$ in the k_0 plane are associated with those of the potential and the form factors. In the QP approximation, the k_0 is usually taken to be constrained to a certain value, depending in general on the 3-momentum \mathbf{k} . Considering the spectator QP approximation [4] (where only the contribution from either the pion or the nucleon pole is taken), we find the somewhat surprising result that condition (4) is not satisfied. The breakdown essentially amounts to that the self-energy does not have the form (2). In particular, 'scalar invariants' like A and B are in this approximation functions of not only P^2 , but also of P_0 . This signals also a violation of the charge conjugation symmetry. The same problem arises in the choices given by Pearce and Jennings[5].

Removing the k_0 singularities (by taking $k_0 = 0$) in the πNN vertex functions, we may perform the k_0 -integration explicitly by contour integration, leading essentially to the ET-approximation. One can readily demonstrate that this choice of QP prescription does satisfy the above discussed symmetry property.

3. A MODEL FOR THE πN SCATTERING

We turn now to the discussion of our model for pion-nucleon scattering. We compute the amplitude by solving the BS equation in the ET approximation. The potential we use is given by the tree diagrams in Figure 1.

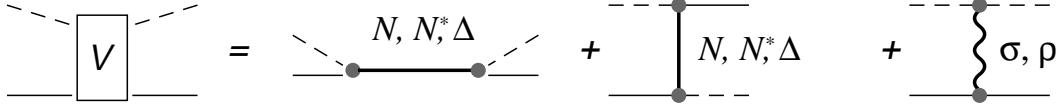


Figure 1. *The model potential.*

We thus include the t -channel σ and $\rho(770)$, and the s - and u -channel $N(938)$, $\Delta(1232)$ and the Roper-resonance exchanges. The Δ and the Roper are widthless when included in the driving force, their one-pion-nucleon decay width is then generated dynamically in the calculation. Note that, although the potential is crossing symmetric, the kernel of the equation and thus the resulting amplitude is not.

The bare vertices and propagators are obtained from an effective Lagrangian of the meson and the isobar fields, see *e.g.* ref.[5]. We allow the pion field to couple only through a derivative coupling, which directly provides the correct low-energy limit, at least at the tree level. The rescattering can in principle violate the low-energy limit (in our model this may come due to the lack of the crossing symmetry). We have checked numerically that these violations are small in our model.

Table 1

The model parameters which were adjusted to reproduce the πN phase-shifts. Only the physical (renormalized) values of parameters are given.

coupling constants	masses [GeV]
$g_{\pi NN}^2/4\pi = 13.5$	$\Lambda_N = 1.5, \Lambda_\pi = 1.5$
$g_{\pi NN^*}^2/4\pi = 0.9$	$\Lambda_{N^*} = 1.9, m_{N^*} = 1.54$
$f_{\pi N\Delta}^2/4\pi = 0.33, z = -0.2$	$\Lambda_\Delta = 1.4, m_\Delta = 1.24$
$g_{\sigma NN}g_{\sigma\pi\pi}/4\pi = 0.1$	$\Lambda_\sigma = 1.1, m_\sigma = 0.55$
$g_{\rho NN}g_{\rho\pi\pi}/4\pi = 2.5, \kappa_\rho = 3.7$	$\Lambda_\rho = 1.1, m_\rho = 0.77$

For each particle we have used a form factor depending on the 4-momentum squared of the particle. For a meson we take the one boson exchange form factor, and for a baryon we use the form factor of Pearce and Jennings[5] (with $n_\alpha = 2$). The corresponding cutoff masses and other model parameters which were fitted to the πN phase-shifts are presented in Table 1.

For the propagator of the Δ we use the Rarita-Schwinger propagator, and the $\pi N\Delta$ coupling is taken to be of the general structure determined by a coupling constant $f_{\pi N\Delta}$ and an off-shell parameter z , cf. ref.[6]. The ρ exchange generates the conserved isovector-vector current. The strength of its coupling is rather close to that determined by the πNN coupling constant through the Kawarabayashi-Suzuki relation ($g_{\pi NN}$ which we use would imply $g_{\rho NN}g_{\rho\pi\pi}/4\pi \approx 2.8$). The ρ thus mainly plays the role of the πN contact term required in the non-linear realizations of chiral symmetry. The σ meson is included so as to simulate the isoscalar-scalar contribution of the correlated two-pion exchange. Therefore, for the $\sigma\pi\pi$ coupling constant we take the sign determined in ref.[7], and we find that this choice is preferable for the proper description of the phase-shifts.

Using this model we are able to get a reasonable fit to the elastic πN phase-shifts,

see Figure 2, with the parameters given in Table 1. To give a feeling about the size of the rescattering contributions in our model, we also show (dashed lines) the results of the calculation where the principal part of the rescattering integrals is neglected (the K-matrix approach).

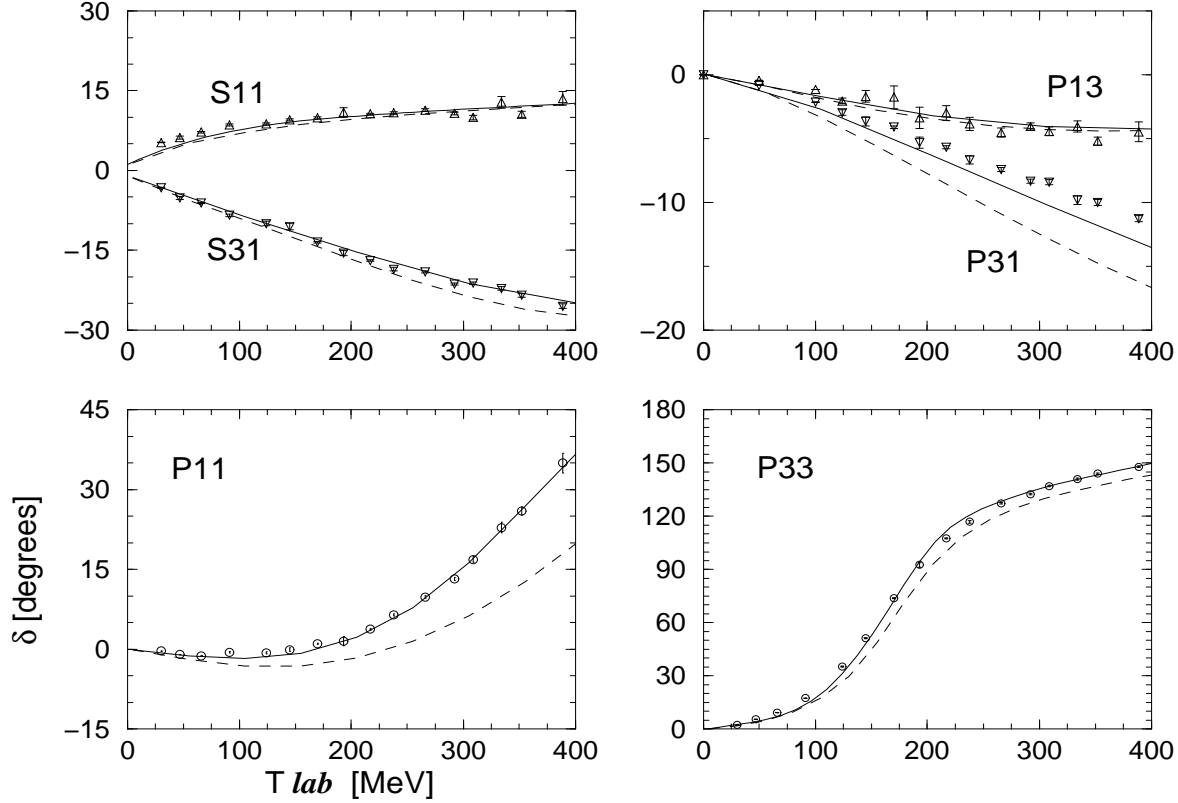


Figure 2. The S- and P-wave πN phase-shifts. Solid lines represent the full calculation, dashed lines are the K-matrix predictions (with the same set of parameters). Data points are from the SM95 partial-wave analysis[8].

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